Hydrodynamic characteristics of transient Ni-like X-ray lasers

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Abstract. A simple similarity model is presented to study hydrodynamics of Transient Collisional Excitation (TCE) nickel-like x-ray lasers. Scaling laws for electron temperature, scale length and electron density are obtained by analytic derivation. The calculations agree well with experimental results.

1. INTRODUCTION

Recently, a novel transient collisional excitation (TCE) scheme was demonstrated by Nickles et al for the Ne-like Ti x-ray laser at 3p-3s (J=0→1) transition of 32.6nm where a high gain of 19 cm⁻¹ was measured with only a few Joule pump energy [1]. More recent results were reported by Dunn et al.[2], where a gain coefficient of 35 cm⁻¹ and a gain-length product of 12.5 were measured on the 4d-4p (J=0→1) transition for the Ni-like Pd laser at 14.7nm using only 5J pump energy. This new scheme has greatly enhanced the efficiency of collisional excitation schemes by increasing the gain coefficient and shows us a way towards “table-top” x-ray lasers.

In this paper, we present a simple similarity model for the TCE Ni-like x-ray lasers and make a detailed analysis using a set of self-similar, coupled ordinary differential equations. The purpose of our work is to understand the hydrodynamic characteristics of transient collisional x-ray lasers and to provide a simple tool for experimentalists to quickly scan and optimize parameters to design experiments.

2. SIMILARITY EQUATIONS AND EQUATION OF STATE

It is well known that the possible gain region for x-ray lasers is in the corona of n_e < n_i and there is an isothermal expansion region during the pulse. Thus a set of self-similarity equations can be obtained from the assumption of rarefactional wave solution of isothermal expansion [3].

\[ LC \left( \frac{d}{dt} \right) \left( \frac{\ln T}{2} + 1 \right) = \frac{pt}{\rho} \]  

(1)

\[ C_v \frac{dT}{dt} = H - \frac{p}{\rho \xi} \]  

(2)

where \( C_v = \frac{dE}{dT} \) is the specific heat per unit mass. The exponential density profile is given by

\[ \rho = m / L \exp(-x / L) \]  

(3)

here m is the mass ablated by the laser pulse from the slab target, L is the scale length of the plasma. With the equation of the mass ablation rate derived by De Groot, et al.[4], we derive the equation of the mass ablation with the assumption of a homogeneous process of the ablation:

\[ m = 2.55 \times 10^{-13} I^{2/3} \left[ \frac{A}{(Z+1)} \right]^{7/6} / (Z \ln \Lambda)^{1/3} t^{2/3} \]  

(4)

here the unit of m is g/cm², the unit of t is ns, the unit of I is W/cm².

The laser heating rate changes with \( \tau_{\text{eb}} \), i.e. \( H \propto I \tau_{\text{eb}} / m \), when \( \tau_{\text{eb}} < 1 \), while the laser heating rate is a constant, i.e. \( H \approx I m \), when \( \tau_{\text{eb}} > 1 \), where \( \tau_{\text{eb}} \) is the inverse bremsstrahlung optical depth through the plasma:

\[ \tau_{\text{eb}} = \int k_{\text{eb}} \, dx \]  

(5)
here $\kappa_{\text{sh}} = \left( \frac{2}{\pi} \right)^{1/2} A \left[ \frac{Zn_e^2 e^6 (\log A) A'}{3 c^3 (m_e T)^{3/2}} \left( 1 - n_e^2 / n_e^2 \right)^{1/2} \right]$ is opacity [5], where $\lambda$ is the laser wavelength, $n_e$ is the electron density, $n_e$ is the critical density ($\approx 1.1 \times 10^{21}/\lambda^3 \text{cm}^3$), $T$ is the electron temperature in energy unit, and $Z$ is the average ion charge.

In the corona region of $n_e \ll n_c$, electron temperature is higher than a few tens eV[6]. Thus we chose

$$Z = \frac{2}{3} \left[ A T, (eV) \right]^{1/3}$$

(6)

where $A$ is the atomic number of the element. And the formula we use for the EOS are

$$p = Z T \rho / M, \quad \text{and} \quad C_e = \frac{3}{2} Z / M, \quad (7)$$

where $M_i$ is the ion mass.

### Table 1. Normalizing values for scaled variables.

<table>
<thead>
<tr>
<th>Physical variable</th>
<th>Time</th>
<th>Laser intensity</th>
<th>Laser wavelength</th>
<th>Ablation mass</th>
<th>Ion charge</th>
<th>Atomic mass</th>
<th>Coulomb logarithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>$T$</td>
<td>$I$</td>
<td>$A_L$</td>
<td>$M$</td>
<td>$Z$</td>
<td>$A$</td>
<td>$A$</td>
</tr>
<tr>
<td>Normalizing value</td>
<td>$\text{ns}$</td>
<td>$10^{19}$Wcm$^{-2}$</td>
<td>$1.053\mu$m</td>
<td>$10^4$gccm$^{-2}$</td>
<td>65</td>
<td>240</td>
<td>5</td>
</tr>
</tbody>
</table>

**3. ANALYTIC SOLUTIONS OF THE SIMILARITY EQUATIONS**

Useful scaling laws for Ni-like scheme have been derived for flat-top laser pulses. We employ convenient units listed in Table I[5] and scale the variable with underline to simplify the calculation.

#### 3.1 Analytic solutions for the long laser pulse

The heating and ionization processes take place for the long pulse duration. The laser energy is mainly deposited near the critical density [6]. Solving the Eq. (1) and (2) with (4), (5), (6) and (7), the analytic solutions, after the influence of the initial conditions has disappeared, i.e. taking $t_{\text{in}} < 1$, are

$$T = 3.902 \text{keV} \cdot I^{5/9} A^{-2/9} \frac{1}{\Lambda^{2/3} I^{2/9}}$$

(8a)

$$L = 2.87 \times 10^{-3} \text{cm} \cdot I^{10/27} A^{-4/27} \frac{1}{\Lambda^{5/27}}$$

(8b)

$$n_e = 20.82 \times 10^{20} \text{cm}^{-3} \cdot I^{11/54} A^{-5/54} \frac{1}{\Lambda^{2/54} I^{14/54}}$$

(8c)

Using Eq. (3), we can determine the electron density at position $x$ and time $t$ with (8c).

#### 3.2 Analytic solutions for the short laser pulse

For the case of "standard" transient collisional x-ray laser, the second pulse is very short. Thus we can assume that: firstly, $m$ is constant after the time $t_{\text{in}}$ and $H=m/m$ because the pre-plasma is not transparent ($t_{\text{in}} > 1$) for the second pulse. Secondly, $Z$ can be taken as a constant because the pulse is too short to change the ionization. The formula we use for the EOS here are Eq.(7).

##### 3.2.1 During the time of $t_{\text{in}} \leq t \leq t_{\text{in}}$

After the first long laser pulse is turned off and the second short laser pulse has started, at time $t_{\text{in}}$, the solutions are obtained by considering the initial conditions before $t_{\text{in}}$:

$$T = 23.123 \text{keV} \cdot I_{2}^{1/2} m^{-1} \Lambda^{-1} I^{1/3} t^{5/3} + \frac{T_{1/2}^{1/2} t_{1/2}^{5/3}}{T_{2}}$$

(9a)

$$L = 5.164 \times 10^{-2} \text{cm} \cdot I_{2}^{1/2} m^{-1/2} I^{3/2} t^{5/3} + \frac{T_{2}^{1/2} t_{1/2}^{5/3}}{L_{2}}$$

(9b)
where $T_{1L}$ is the electron temperature at $t_{1L}$, $T_{2} = 23.123$ keV, $L_{2} = 5.164 \times 10^{-2}$ cm, $n_{1L}$ is the density at $t_{1L}$, $n_{2} = 3.157 \times 10^{20}$ cm$^{-3}$.

### 3.2.2 During the time of $t_{1L} \leq t$

After the time $t_{1L}$, the second short laser pulse is turned off and the plasma continues to expand adiabatically. The exact analytical solutions are obtained for this period using the condition before $t_{1L}$:

$$T = 23.123 \text{ keV} L_{2} m^{-1} A Z^{-1} \left(1 - t_{1L}^{-5/3} t_{2L}^{-5/3} + \frac{n_{2}^{2} L_{2}^{2/3} Z^{-5/3}}{n_{1L}^{2} L_{1L}^{2/3}}\right) \left(1 - t_{1L}^{-5/3} t_{2L}^{-5/3} \right)$$

(10a)

$$L = L_{2} t_{2L}^{-5/6} \left(1 - t_{1L}^{-5/3} t_{2L}^{-5/3} + \frac{n_{2}^{2} L_{2}^{2/3} Z^{-5/3}}{n_{1L}^{2} L_{1L}^{2/3}}\right)^{1/2} \left(1 - t_{1L}^{-5/3} t_{2L}^{-5/3} \right)$$

(10b)

$$n_{0} = n_{2}^{2} t_{2L}^{-5/6} \left(1 - t_{1L}^{-5/3} t_{2L}^{-5/3} + \frac{n_{2}^{2} L_{2}^{2/3} Z^{-5/3}}{n_{1L}^{2} L_{1L}^{2/3}}\right)^{-1/2} \left(1 - t_{1L}^{-5/3} t_{2L}^{-5/3} \right)$$

(10c)

### 4. RESULTS AND DISCUSSION

In order to show the reliability of the model, we calculate the hydrodynamics of the transient collisional Ni-like Pd x-ray laser under the same conditions from Dunn’s experiment [2].

The first goal for the long pulse is to produce a high abundance of Ni-like ions. A longer constant temperature period is of course, beneficial for the production of a sort of ions from Eq.(6). From Eq.(8a), the temperature is almost a constant at the later time. Thus, the longer the pulse duration, the closer to a constant temperature the temperature is (see Fig.1 (a)). Simultaneously, the longer pulse duration can also make a longer scale length (see Fig.1 (b)), which is beneficial for the propagation of x-ray laser.

The second goal is to enhance efficiency. It will be highly efficient when the long pulse not only ionize the plasma to be Ni-like ions, but also achieve a thermodynamic equilibrium between electrons and ions. The time of the thermodynamic equilibrium is $t_{e} = \left(4 \pi \varepsilon_{0} e^{-2}/3 m_{e} \left(k T_{e}^{3/2}\right) \sqrt{2\pi Z^{2} n_{e} \left(\log \Lambda\right)}\right]$. It can also be used to judge whether the short pulse duration is short enough to ignore the change of $Z$.

We calculated the Ni-like Pd ions for the long pulse under the same conditions of Dunn’s (1.7x10$^{12}$ W/cm$^2$, 1.053$\mu$m, t$_{1L}$=0.8 ns, t$_{2}$=5.2x10$^{14}$W/cm$^2$, t$_{1L}$=1.1 fs). The dashed line is the solution for $I_{1}=0.7x10^{12}$ W/cm$^2$, $\lambda_{1}=1.053 \mu$m, $t_{1L}=0.8$ ns, slab. The results show that the maximum electron temperature during the long pulse is 188 eV, which can ionize the Pd to an average state of charge $Z=18.1$, see the solid line in Fig.1(a). The scale length is about 40$\mu$m, as the solid line shows in Fig.1(b). The electron density is 7.5x10$^{20}$ cm$^{-3}$, see the solid line in Fig.1(c). However, the equilibrium time is about 1.4 ns. The $t_{1L}/t_{e}$ is about 0.57. For comparison, we also calculated the electron temperature and the average state of charge...
with the conditions of $I_p=0.6 \times 10^{12}$ W/cm$^2$, $\lambda=1.053$ nm, $t_{fs}=1.2$ ns, slab and the same short pulse. The results are $T_e=188.8$ keV, $Z=18.1$, $L=60$ nm, $n_e=7.28 \times 10^{20}$ cm$^{-3}$, shown by the dashed line in Fig.1, the equilibrium time is about 1.8 ns. The $t_{eq}/t_{el}$ is about 0.67. For the short laser pulse, the temperature is 1.84 keV under Dunn's condition and 2.07 keV in the second case. The equilibrium times are about 55.5 ns and 66.2 ns, respectively. The $t_{eq}/t_{el}$ are about 1.98 $\times 10^2$ and 1.66 $\times 10^2$, respectively. It is apparent that the pulse duration for the long pulses is shorter than the equilibrium time for both cases. However, It is more efficient for the second case than Dunn's and the scale length is longer than Dunn's. The conclusion here is that the lower intensity and longer duration of the long laser pulse will be more beneficial for the efficiency. The electron density history at the different positions for the two cases is shown in Fig.1(d).

In contrast with the long pulse, the first important thing for the short pulse is that $T_e$ can not be lower than $\Delta E_e$ from Dunn, et al.[2], where $\Delta E_e$ is the upper laser level excitation energy. Here $\Delta E_e=450.3$ eV. The results show that the electron temperature during the short pulse period is 1.84 keV under Dunn's conditions ($I_p=5.2 \times 10^{14}$ W/cm$^2$, $\lambda=1.053$ nm, $t_{fs}=1.1$ ps, slab), which is about 4 times as $\Delta E_e=450.3$ eV, and is heated rapidly, shown by the solid line in Fig.1(a). The second important thing is to enhance the gain coefficient, and a shorter pulse is better to enhance the gain coefficient since it heats not only the electron temperature up rapidly but also keep the ion temperature constant. This will be advantageous to enhance the gain coefficient because it is proportional to the electron temperature and inversely proportional to the square root of the ion temperature.

![Figure 2](image)

**Figure 2** Temperature history (a), scaling length history (b), density curves with $x$ (c) and Temperature curve with pulse duration (d) from the similarity equations for different duration of a flat-top pulse. The solid line is the solutions of long pulse. The dashed and dash-dot lines are the solutions for the different short pulse duration with same intensity at $t_{fs}=1.1$ ps and $t_{fs}=110$ fs, respectively. Where $I_p=0.7 \times 10^{11}$ W/cm$^2$, $\lambda=1.053$ nm, $t_{el}=0.8$ ns, $I_p=5.2 \times 10^{14}$ W/cm$^2$.

In order to show the hydrodynamic characteristic of the plasma for the short pulse, We investigated the relations of electron temperature, scale length and electron density with different pulse duration. The result shows that the electron temperature increases rapidly from 330 eV to 1.84 keV with the increase of pulse duration from 110 fs to 1.1 ps, shown in Fig.2(a) and 2(d). In the same time, the scale length increases from 50 to 80 μm, which makes the density gradient more relaxed for the longer pulse than shorter one, see in Fig.2(b) and (c). The electron density is $7.5 \times 10^{20}$ cm$^{-3}$. It is clear that too short pulse is not beneficial for the production of x-ray laser under the same intensity.

5. CONCLUSIONS

In conclusion, we have developed a similarity model for transient x-ray lasers. The calculated results are in agreement with Dunn's experimental results. The model is useful for an approximate, quick parameter scan for the experimental design and analysis.

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Reference