

Elasto-Plastic Micro-Macro Modelling of Solid-Solid Phase Transformation; Application to Transformation Induced Plasticity

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Abstract : Classical plasticity (dislocation motion) coupled to deformation due to lattice changes (phase transformation) occurs in several materials where interfaces are moving under external forces (TRIP (TRansformation Induced Plasticity) steels, even shape memory alloys). In the case of steels (transformation induced plasticity) the internal stress associated with the phase change induces a large additional plastic flow inside austenite and martensite. We propose a micromechanical modelling of such a phenomenon based on a decomposition of strain rate into an elastoplastic part and a given lattice inelastic strain rate field. Using usual Green functions method, a concentration tensor for the total strain rate field is obtained. The self consistent approximation allows to determine the behavior of the equivalent material and the equivalent transformation strain rate. Applications in the case of cooling under constant applied stresses and isothermal loading give results in good agreement with Finite Element calculations and theoretical results.

1. INTRODUCTION

In the case of TRIP steel the determination of the global behavior during phase transformation is more complicated than in the case of shape memory alloys since the response of the material to an external thermomechanical loading is not only due to a large scale orientation of the transformation strain of martensite [1] but also to a large scale orientation of plastic flow in austenite [2] and martensite due to irreversible strains at the microscopic scale. The aim of this paper is to clarify these different points with a new rate approach of the problem.

In the last ten years most of the authors [3][4][5][6] have used, for numerical simulations, a phenomenological law which gives the macroscopic transformation strain (additional macroscopic strain E^{TR} , see [7]) or strain rate (\dot{E}^{TR}) versus the volume fraction of martensite f (two-phase steel) that is :

$$\dot{E}_{ij}^{TR} = C S_{ij} g(f) \dot{f}$$

where C is a scalar, S_{ij} is the deviatoric part of the macroscopic stress Σ_{ij} . The justification of such a law will be referred here as the **first problem**.

In all cases the authors do not deal in general with the **second problem** which divides into two questions:

- to express the law which gives the volume fraction f of martensite versus loading (controlling external parameters (stress or strain and temperature) :

$$\dot{f} = ()\dot{\Sigma} + ()T$$

Only [8] made an attempt in this direction based on physical assumptions refined later [9].

- to give the mean value of the transformation strain (or strain rate) over all the different martensite variants :

$$\bar{\varepsilon}^B(t) = \frac{1}{V} \int_V \dot{\varepsilon}^B(r,t) ds = (\dot{\Sigma} + \dot{T})$$

that is to solve the problem posed by [1].

The complete resolution of a structure undergoing a martensitic transformation is therefore an open question since it requires the knowledge of the oriented volume fraction of martensite versus external loading (and some internal parameters too!) at a microscopic scale and the mean value of the transformation strain over these martensite variants ; this so-defined **second problem** will not be treated here.

We shall focused here on the **first problem** where f and $\bar{\varepsilon}^B$ are supposed to be known ; it has been already treated in a secant moduli approximation [10] and refined in a rate formulation [11] using a self-consistent approach. Here only the basis of the modelling are recalled and used to explain the response of incompressible and isotropic materials to several thermomechanical loading paths.

2. MICROMECHANICAL MODELLING

2.1. General problem

We consider a macroscopic element of matter (V) containing two phases. Under a thermomechanical loading one part (previously austenite) has been transformed into martensite, i.e. a lattice transformation has occurred defined locally (and completely) by the deformation ε^{B_i} and the rotation ω^{B_i} in the i^{st} variant or lath of martensite. The deformation ε^{B_i} is called the Bain strain. The microscopic total strain is assumed to be small and can therefore be decomposed into elastic, plastic and Bain strain as :

$$\varepsilon^i(r,t) = \varepsilon^e(r,t) + \varepsilon^p(r,t) + \sum_I \varepsilon^{B_i} \theta^I(r,t) = e(r,t) + \varepsilon^B(r,t) \quad (1)$$

where $e(r,t) = \varepsilon^e(r,t) + \varepsilon^p(r,t)$ and $\theta^I(r,t)$ is defined by :

$$\theta^I(r,t) = \begin{cases} 0 & \text{if } r \notin V_m^I \\ 1 & \text{if } r \in V_m^I \end{cases} \quad (2)$$

where V_m^I is the i^{st} martensitic volume. The problem governing equations are then :

- quasi-static equilibrium (no volumic forces) :

$$\dot{\sigma}_{ij,j}(r) = 0 \quad (3)$$

- boundary conditions (velocities for example) :

$$v_i = \dot{E}_{ij} x_j \quad (4)$$

- kinematic relations :

$$v_{i,j} = \dot{e}_{ij} + \dot{\varepsilon}_{ij}^B \quad (5)$$

- elasto-plastic constitutive equations :

$$\dot{\sigma}_{ij}(r) = l_{ijkl}(r,e) \dot{e}_{kl}(r). \quad (6)$$

In addition usual assumptions of symetries for l_{ijkl} are made, which are :

$$l_{ijkl} = l_{jikl} = l_{ijlk} = l_{klij} \quad (7)$$

2.2. General equations in the case of incompressible isotropic materials

Using a selfconsistent approach together with a local approximation detailed in [11] one finds the governing equations for the previous problem. In the case of incompressible isotropic materials with a uniform behavior in each phase these equations reduce to :

$$\dot{\Sigma}_{xx} = 3\mu_e(\dot{E}_{xx} - \dot{E}_{xx}^{TR}) \quad (8)$$

for an uniaxial loading (compression or tension), where $\dot{\Sigma}_{xx}$ is the macroscopic stress rate in the x direction. The effective tangent shear modulus μ_e is given by :

$$f \frac{5\mu_e}{3\mu_e + 2\mu_m} + (1-f) \frac{5\mu_e}{3\mu_e + 2\mu_a} = 1 \quad (9)$$

where μ_a and μ_m are the tangent shear moduli in austenite and martensite (which are supposed to be known) and f the volume fraction of martensite. The macroscopic transformation strain rate in the x direction, \dot{E}_{xx}^{TR} , is given by :

$$\dot{E}_{xx}^{TR} = \frac{5\mu_m}{3\mu_e + 2\mu_m} \dot{f} \bar{\epsilon}_{xx}^B \quad (10)$$

One can compare this expression of the macroscopic transformation strain rate with that given in the introduction and referred as the first problem. Since $\mu_e \leq \mu_m$ in general (martensite is the hardest phase in steels) one can see that $\dot{E}_{xx}^{TR} \geq \dot{f} \bar{\epsilon}_{xx}^B$ which can be seen as a generalization of the TRIP effect - as introduced in [2]. In this formula $\bar{\epsilon}_{xx}^B$ is the mean Bain strain over a representative domain of surface S with unit normal \vec{n} and interphase velocity \vec{W} :

$$\bar{\epsilon}_{xx}^B = \frac{\int_S \epsilon_{xx}^B \vec{W} \vec{n} ds}{\int_S \vec{W} \vec{n} ds} \quad (11)$$

The rate of the volume fraction of martensite \dot{f} is given by :

$$\dot{f} = \frac{\dot{V}_m}{V} = \frac{1}{V} \int_S \vec{W} \vec{n} ds \quad (12)$$

Equations (8), (9) and (10) allow to determine the overall behavior in various (uniaxial) situations where μ_a , μ_m , $\bar{\epsilon}_{xx}^B$ and \dot{f} are given.

3. ANALYTICAL RESULTS AND DISCUSSION

3.1. Cooling under constant macroscopic applied stress

In a lot of experimental studies of transformation plasticity, the stress applied during the transformation is kept constant (the registered strain is then the macroscopic transformation strain E^{TR}),

small (notably lower than the yield stress σ_a^Y in the weakest phase, here austenite) and uniaxial, the specimen being cooled so to study the influence of f .

Due to the small stresses, authors [12] usually assume that the martensite remains elastic ($\mu_m = \mu^{el}$) while austenite is plastifying (because of the transformation). In this case one can easily find μ_e by solving equation (9) assuming $\mu_a \ll \mu^{el}$, where μ^{el} is the elastic modulus of martensite ; it comes:

$$f \ll 0,4 : \frac{\mu_e}{\mu^{el}} = \frac{2}{2-5f} \frac{\mu_a}{\mu^{el}} \tag{13a}$$

$$f \gg 0,4 : \frac{\mu_e}{\mu^{el}} = \frac{5f-2}{3} \tag{13b}$$

for $f \approx 0,4$ only a complete solution can be given. In the case of an ideal-plastic austenite ($\mu_a = 0$) it finally comes after integration of (10) :

$$f < 0,4 : E_{xx}^{TR} = \frac{5}{2} \int_0^f \bar{\epsilon}_{xx}^B(y, \Sigma_{xx}) dy \tag{14a}$$

$$f > 0,4 : E_{xx}^{TR} = \bar{\epsilon}_{xx}^B(0,4, \Sigma_{xx}) + \int_{0,4}^f \frac{\bar{\epsilon}_{xx}^B(y, \Sigma_{xx})}{y} dy \tag{14b}$$

Figure 1 shows analytical $E_{xx}^{TR}(f)$ curves obtained for a A.508 steel [12], whose characteristics are $\sigma_a^Y = 145MPa$, $\sigma_m^Y = 950MPa$ with both phases ideal plastic.

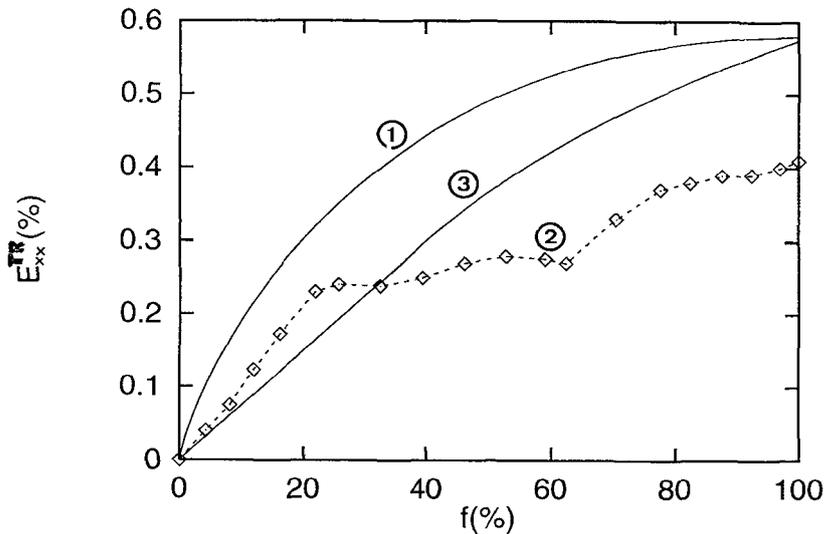


Figure 1 : transformation plastic strain for a stress equal to 50 MPa : ① : theoretical model [12] ; ② : Finite Element calculations [12] ; ③ : selfconsistent model with $\bar{\epsilon}_{xx}^B(f, \Sigma_{xx}) = 0,3\%$ constant

In spite of very crude approximations (leading to enlightening analytical equations) one can see on fig. 1 the good agreement of this model with theoretical and FE calculations as well as the resulting TRIP effect : $E_{xx}^{TR} > f \bar{E}_{xx}^B$.

3.2. Isothermal stress-strain curves. Dynamic softening

The model is used here to describe the dynamic softening which appears in metastable austenitic steels exhibiting TRIP effect at the beginning of the inelastic macroscopic flow (see figure 2).

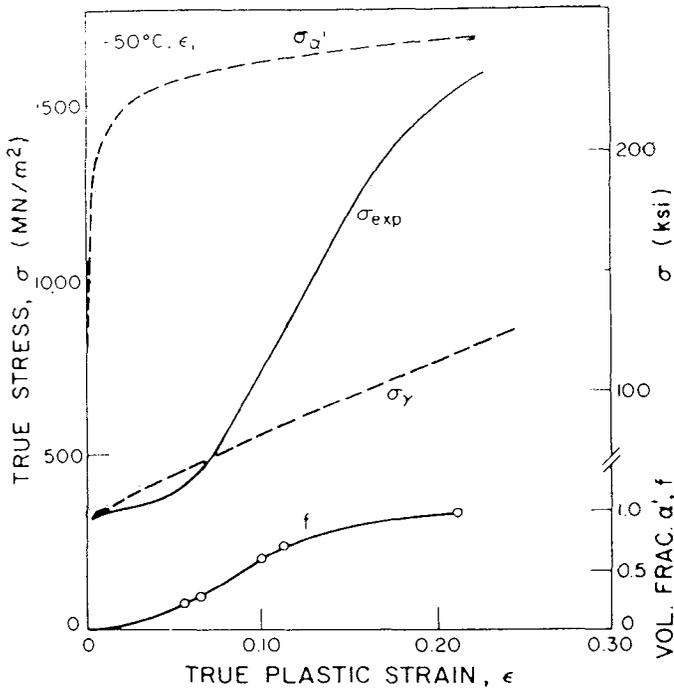


Figure 2 : experimental macroscopic flow stress (tensile test) and volume fraction of martensite vs plastic strain for a metastable austenitic steel from [13]. Dashed curve represents the stable austenite and martensite flow stress

As long as the austenite is elastic, that is for $E_{xx} < \frac{\sigma_a^y}{3\mu^{el}}$, no martensite is supposed to appear (one doesn't deal here with stress-assisted martensite). For $E_{xx} \geq \frac{\sigma_a^y}{3\mu^{el}}$, that is for $f \geq 0$, one shall now compare the ratio $\frac{\dot{\Sigma}_{xx}}{E_{xx}}$ which represents the macroscopic hardening to $3\mu_a$ which describes the hardening in the austenite. Considering that martensite remains elastic ($\Sigma_{xx} \approx \sigma_a^y$) and assuming the ratio $\frac{\mu_a}{\mu^{el}}$ to be small (see figure 2) as well as the amount of martensite (only the beginning of the stress-strain curve is of interest here) equations (9) and (10) reduce to :

$$\mu_e = \frac{2}{2-5f} \mu_a \quad \dot{E}_{xx}^{TR} = \frac{5}{2} \bar{E}_{xx}^B f \quad (15), (16)$$

The ratio

$$\frac{\dot{\Sigma}_{xx}}{\dot{E}_{xx}} \approx 3\mu_a \frac{1 - \frac{5}{2} \frac{\bar{E}_{xx}^B \dot{f}}{\dot{E}_{xx}}}{1 - \frac{5}{2} f} \quad (17)$$

gives the slope of the dynamic softening at the beginning of the strain-induced transformation. Since $\frac{\bar{E}_{xx}^B}{\dot{E}_{xx}}$ is always positive as well as \dot{f} , $\frac{\dot{\Sigma}_{xx}}{\dot{E}_{xx}}$ can be less than $3\mu_a$ according to the relative values of \dot{E}_{xx} and $\bar{E}_{xx}^B \frac{\dot{f}}{f}$, so that the model may include dynamic softening.

4. CONCLUSION

In spite of strong approximations leading to the simple analytical results presented here :

- Two phase approximation
- Incompressibility and isotropic behavior
- First order simplification giving analytical results (elastic martensite)

the model allowed to reproduce qualitatively and quantitatively the main effects of solid phase transformation on the behavior of TRIP steels :

- TRIP effect
- dynamic softening

The next step of the modelling is to describe the transformation itself (so-called **second problem**).

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